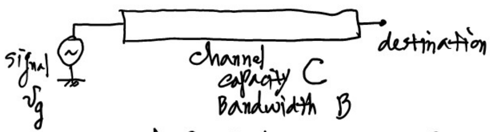
Shannon's Limit for Energy in Transmitting One bit of Information



rate of signal transmission R

N_o = noise energy per unit bandwidth

E_b = average energy per 'bit' delivered to output (destination)

Signal to Noise Ration (SNR) $= \frac{E_b \cdot R}{N_o \cdot B}$

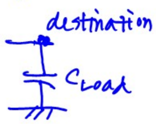
Shannon's Theorem

A channel that is communicating with the use of a signal bandwidth B and a signal-to-noise ratio (SNR) has a channel capacity C given by

$$C = B \log_2 (1 + SNR) \qquad (1)$$

$$SNR = \frac{\overline{v_{sig}^2}}{\overline{v_n^2}} \qquad (2)$$

where $\overline{v_n^2} = \frac{kT}{C_{Load}}$



$k = 1.38 \times 10^{-23}$ J/K

$T = 300°K$ at room temp.

circuit example

Noise energy per unit bandwidth

$$N_o = \frac{1}{2} C_{Load} \overline{v_n^2}$$

$$\overline{v_n^2} = \frac{kT}{C_{Load}} \quad \left(\frac{1}{2} kT\right)$$

$$E_b = \frac{1}{2} C_{Load} \overline{v_{sig}^2}$$

$$SNR = \frac{E_b \cdot R}{N_o \, B}$$

$$E_b = SNR \frac{N_o B}{R} = SNR \frac{N_o}{\left(\frac{R}{B}\right)}$$

per shannon's theorem eq (1)

$$C = \log_2 (1 + SNR)$$

$$\frac{R}{B} < C = \log_2 (1 + SNR)$$

$$\frac{R}{B} < \log_2 (1 + SNR)$$

$$2 \cancel{e}^{\frac{R}{B}} < \underset{2}{\cancel{e}}^{\log_2 (1 + SNR)} = 1 + SNR$$

$$\Rightarrow 2^{\frac{R}{B}} < 1 + SNR \Rightarrow SNR > 2^{\frac{R}{B}} - 1$$

$$E_b = SNR \frac{N_o}{\frac{R}{B}}$$

$$> \left(2^{\frac{R}{B}} - 1\right) \frac{N_o}{\left(\frac{R}{B}\right)}$$

when $\frac{R}{B} \ll 1$

$$\lim_{\frac{R}{B} \to 0} E_b > \underbrace{\lim_{\frac{R}{B} \to 0} \left(\frac{2^{\frac{R}{B}} - 1}{\frac{R}{B}} \right)}_{= \ln 2 = 0.69} \overset{\frac{1}{2} kT}{\overset{\parallel}{N_o}}$$

$$\boxed{E_b > \frac{1}{2} kT \cdot \ln 2}$$

$$\lim_{\frac{R}{B}\to 0}\left(\frac{2^{\frac{R}{B}, x}-1}{\frac{R}{B}, x}\right)$$

when $\frac{R}{B}=0$, $\frac{2^0-1}{0}=\frac{0}{0}$

L'Hospital's rule

$$\frac{\frac{d}{dx}\left(2^x-1\right)}{\frac{d}{dx}x}=\frac{\frac{d}{dx}\left(2^x\right)-0}{1} \quad = \ln 2$$

$$a^x=(\underset{\overset{\shortparallel}{a}}{e^{\ln a}})^x=e^{x\ln a}$$

$$\frac{d}{dx}\left(a^x\right)=\frac{d}{dx}\left(e^{x\ln a}\right)$$

$$=\ln a\, e^{x\ln a}$$

$$=\ln a\,\underbrace{e^{\ln a^x}}_{a^x}$$

$$=\ln a\cdot a^x$$

when $a=2$,

$$\left.\frac{d}{dx}\left(2^x\right)\right|_{x=0}=\left.\ln 2\ 2^x\right|_{x=0}=\ln 2$$